Comparison of different solution techniques of the 2-D Steady State Inverse Heat Conduction Problem

Fabio Bozzoli, Sara Rainieri, Giorgio Pagliarini

Department of Industrial Engineering, University of Parma, Italy

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Introduction

- The solution of the **2-D Steady state Inverse Heat Conduction Problem** (SIHCP), by using the surface temperature distribution as input data, enables to estimate the **local heat transfer coefficient**, an important parameter in heat exchangers design.

- Although **SIHCP** can be regarded as a special case of the Unsteady state Inverse Heat Conduction Problem , some caution is needed in the application of standard inverse solution strategies to this particular problem: **destructive effect of noise which is amplified** in the steady case by the necessity of estimating the wanted information from the signal Laplacian and not from the signal first temporal derivative.

$$\lambda \nabla^2 T(x,y) + q_g = \rho \cdot c_p \frac{\partial T}{\partial t}$$

Introduction

In the present work a **simulated noisy signal**, representing the experimental input data of the SIHCP with convective thermal boundary conditions, is considered in order to compare different solution techniques.

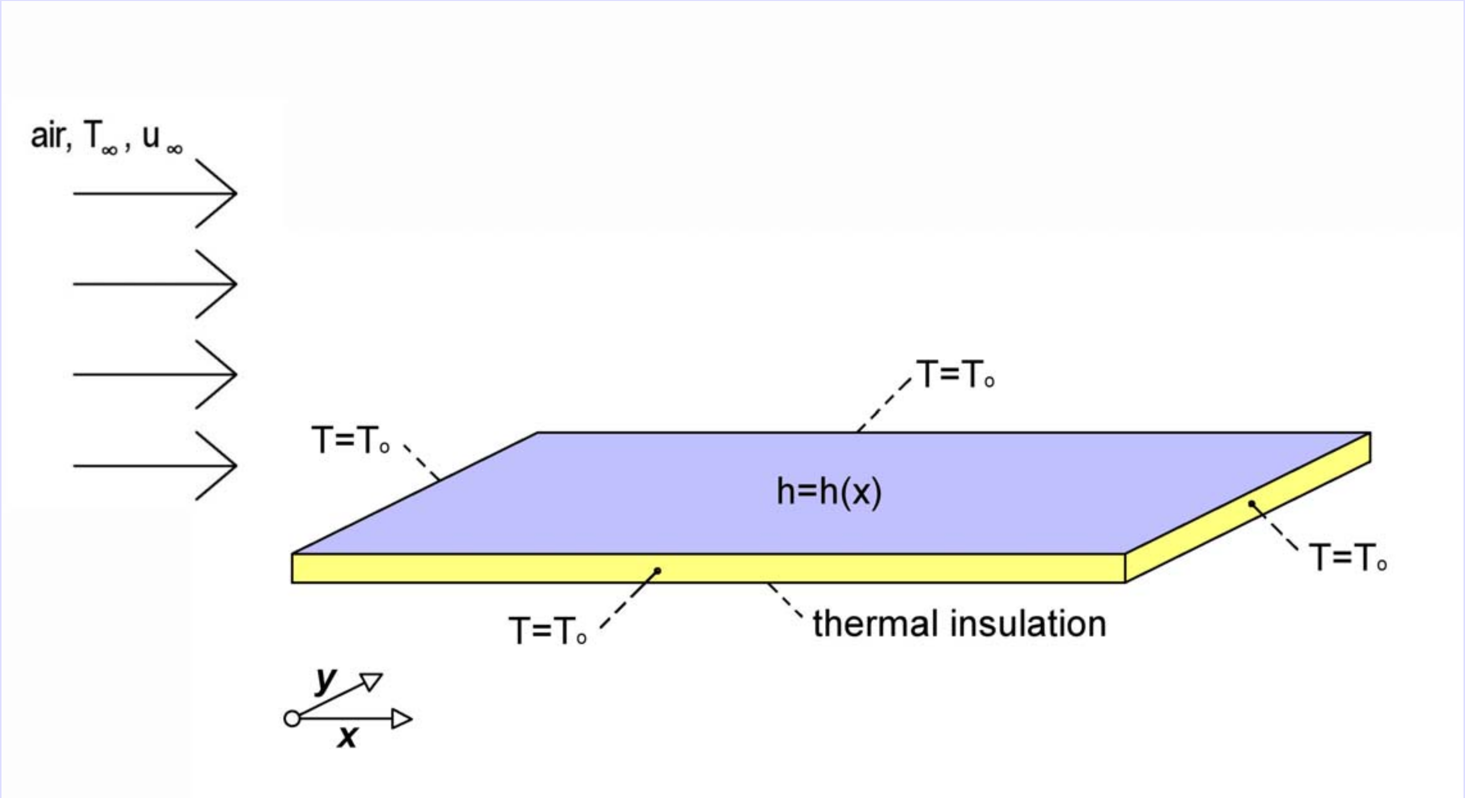
In particular the temperature distribution occurring on aluminum plate, exposed on one side to a forced turbulent air flow, is simulated.

INPUT
the temperature map

↓

UNKNOWN
local heat transfer

Test case

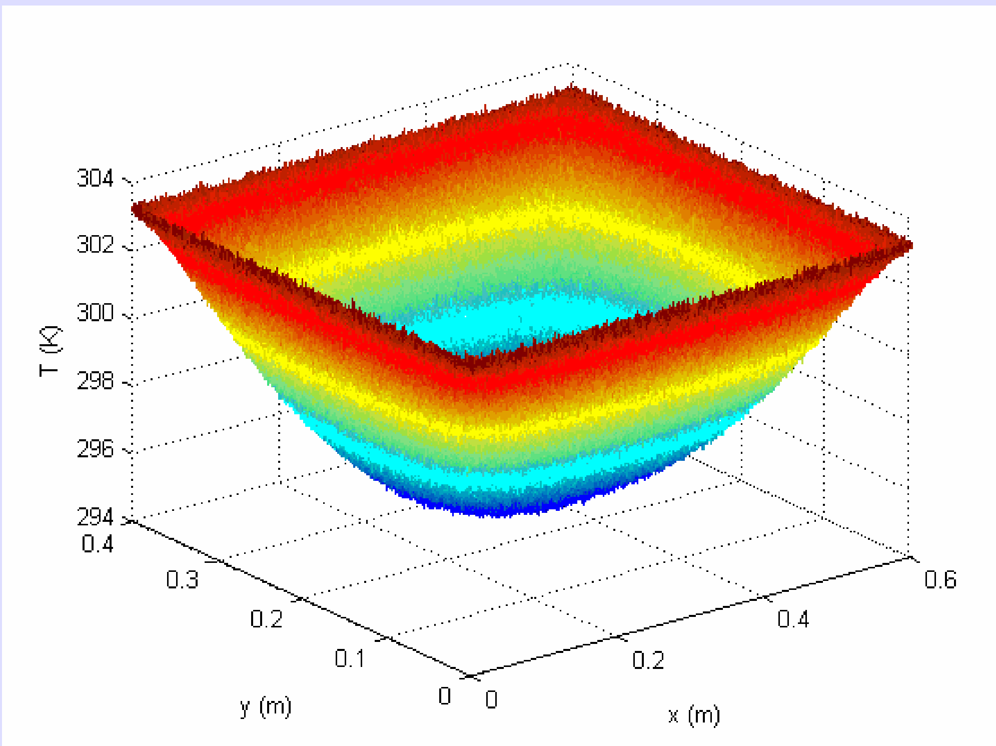


$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = \frac{g(x,y)}{s \cdot \lambda}$$

$$h(x,y) = \frac{g(x,y)}{T(x,y) - T_\infty}$$

Test case

In particular:
600mm x 400mm x 3mm aluminum plate, ΔT = 10 K,
turbulent flow convection (*h* shows a moderate dependence from x)



- The signal has been mapped on a discrete domain with 600×400 equally spaced elements.
- A random additive noise with a standard deviation of 0.1 K has been imposed.

(thermographic systems)

Solution techniques

Among the techniques available in literature for the solution of inverse heat transfer problems, these ones appear the most suitable for this particular application:

- the “forced matching technique”;
- the “Wiener filtering technique”;
- the “Conjugate Gradient Method with adjoint problem formulation”.

Forced matching

The temperature distribution obtained by numerically solving the corresponding direct problem, is forced to match the experimental noisy data.

$$h^{n+1}(x,y) = \frac{h^n(x,y)\cdot(T^n_{NUM}(x,y) - T_\infty)}{T_{EXP}(x,y) - T_\infty}$$

where $\boldsymbol{T^n_{NUM}}$ is temperature distribution obtained by solving numerically the conduction problem imposing $\boldsymbol{h^n_{tot}}$ as a convective heath transfer coefficient; while $\boldsymbol{T_{EXP}}$ is the experimental temperature map.

Wiener filtering

Consisting in **two consecutive applications of the Wiener filter**, removes from the raw temperature data the unwanted experimental noise by making the **direct calculation of the signal’s Laplacian feasible**.

The filtering function uses a pixel-wise adaptive filtering procedure based on statistics estimated from a local neighborhood of each data point.

$$\mu = \frac{1}{NM} \sum_{i,j \in \eta} T_{EXP}(i,j)$$

$$\sigma^2 = \frac{1}{NM} \sum_{i,j \in \eta} T_{EXP}{}^2(i,j) - \mu^2$$

$$T_W(i,j) = \mu + \frac{\sigma^2 - \upsilon^2}{\sigma^2}\left(T_{EXP}(i,j) - \mu\right)$$

For a successful application, it is necessary to adjust the range of action of the filter, defined by the size of the pixel's neighborhood η.

Conjugate Gradient Method with adjoint problem formulation

- CGM handles the ill-posed nature of the problem by reformulating the problem as a well-posed problem by **minimizing the squared difference** between measured and estimated temperature discrete data.

$$J\left[g(x,y)\right]=\sum_{i=1}^{M}\left[T(X_i,Y_i)-T_{EXP}(X_i,Y_i)\right]^2$$

- It is a **gradient-based optimization procedure**: it searches iteratively in this direction the minimum of the objective function.

- Under the adjoint equation approach, such minimization procedure requires the solution of **auxiliary problems**, known as the **sensitivity** and the **adjoint** problem.

Conjugate Gradient Method with adjoint problem formulation

sensitivity problem

$$\frac{\partial^2 \Delta T(x,y)}{\partial x^2} + \frac{\partial^2 \Delta T\ (x,y)}{\partial y^2} = \frac{q^k(x,y)}{s \cdot \lambda}$$

$$\Delta T(x,y) = 0 \qquad (x,y) \in \partial D$$

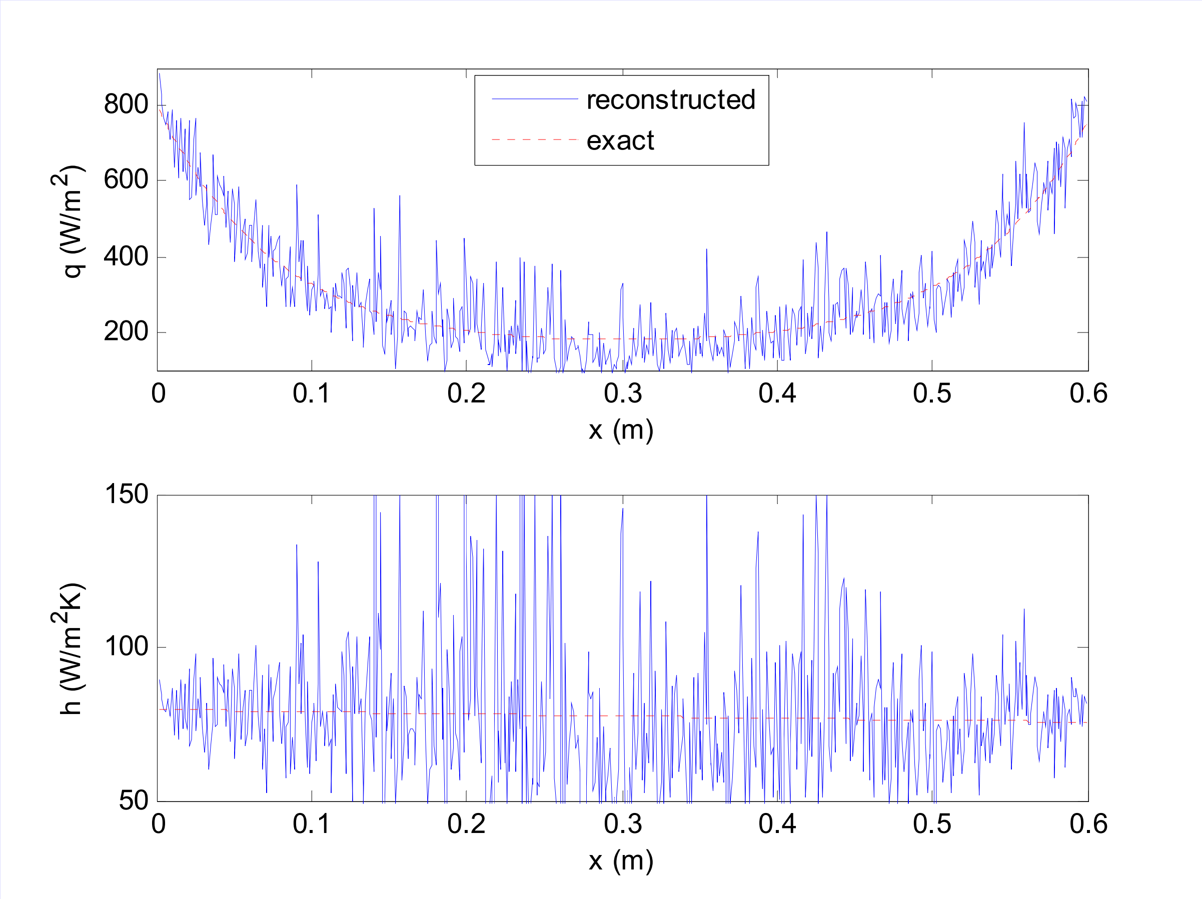
adjoint problem

$$2 \cdot \left[T_{EXP}(x,y) - T(x,y) \right] = \frac{\partial^2 J'(x,y)}{\partial x^2} + \frac{\partial^2 J'(x,y)}{\partial y^2}$$

$$J'(x,y) = 0 \qquad (x,y) \in \partial D$$

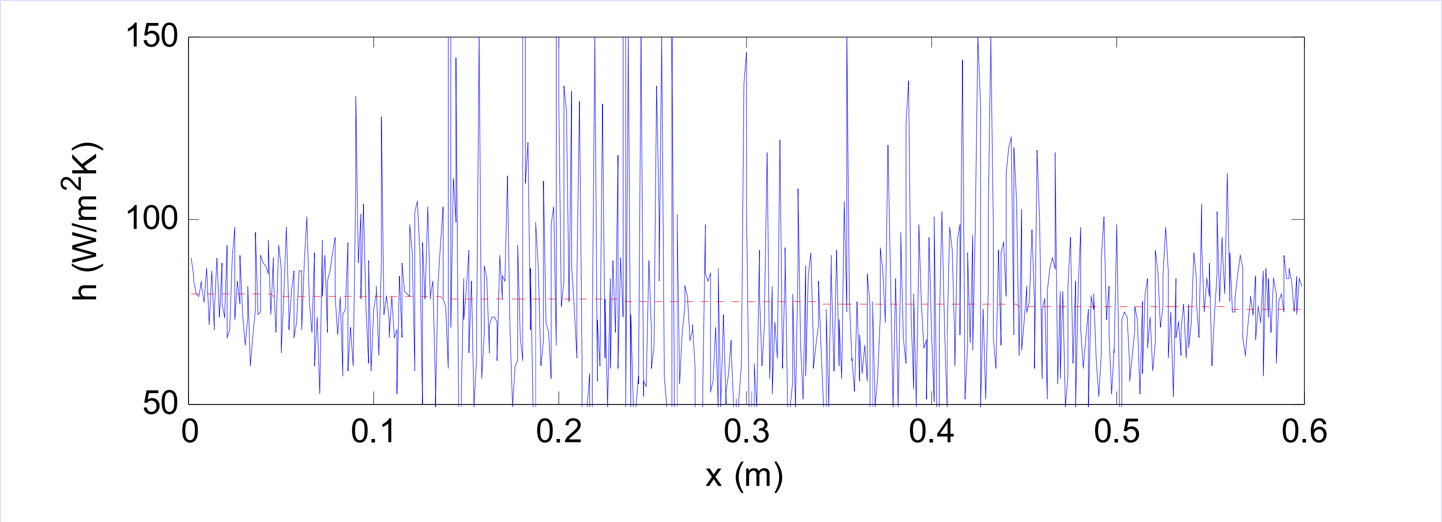
stopping criterion: discrepancy principle

Results: Forced matching



Surface heat flux and convective heat transfer coefficient distribution along the domain centerline.

Results: Forced matching

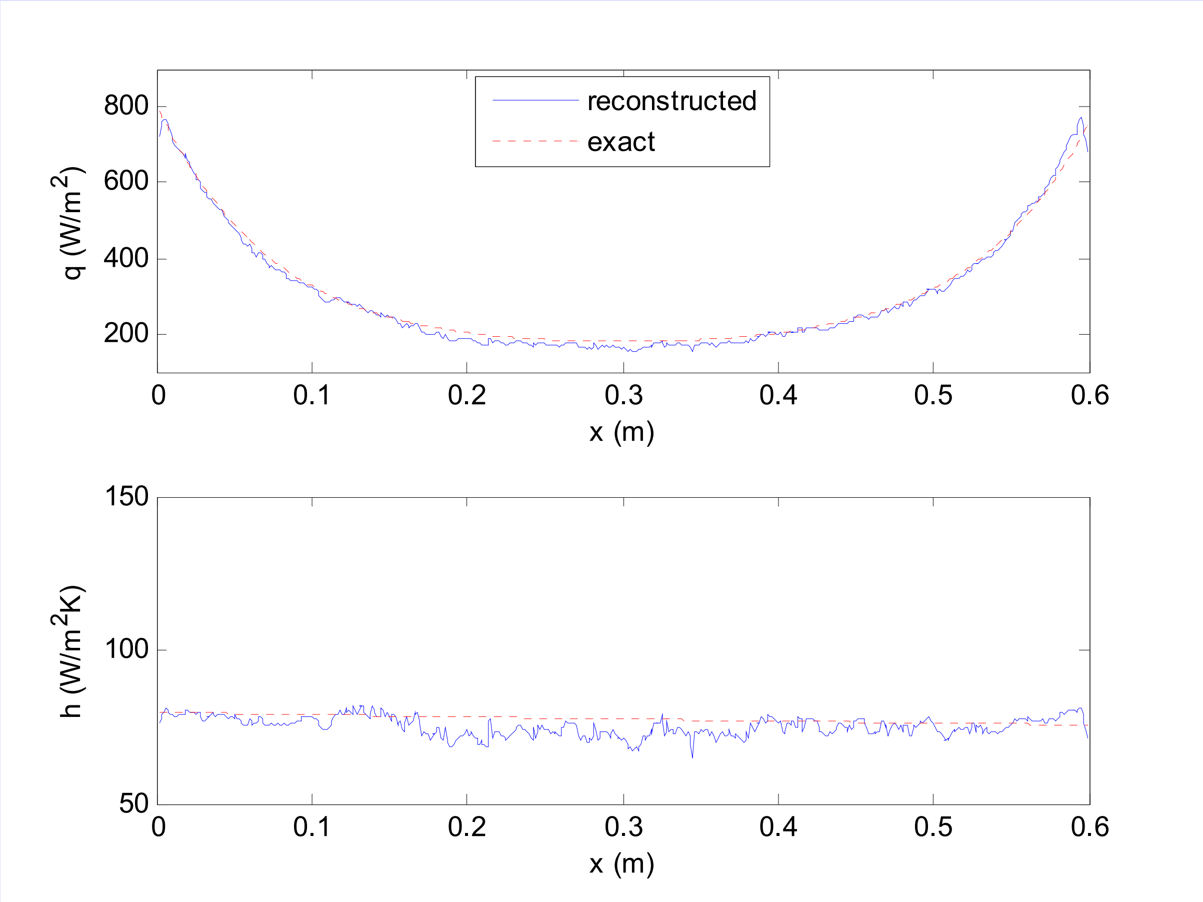


➢ **PROS**: easy to implement, requires limited computational resources, effective also with a low signal-to-noise ratio.

➢ **CONS**: a poor effectiveness in terms of local estimation capability (although the average values are restored with sufficient accuracy).

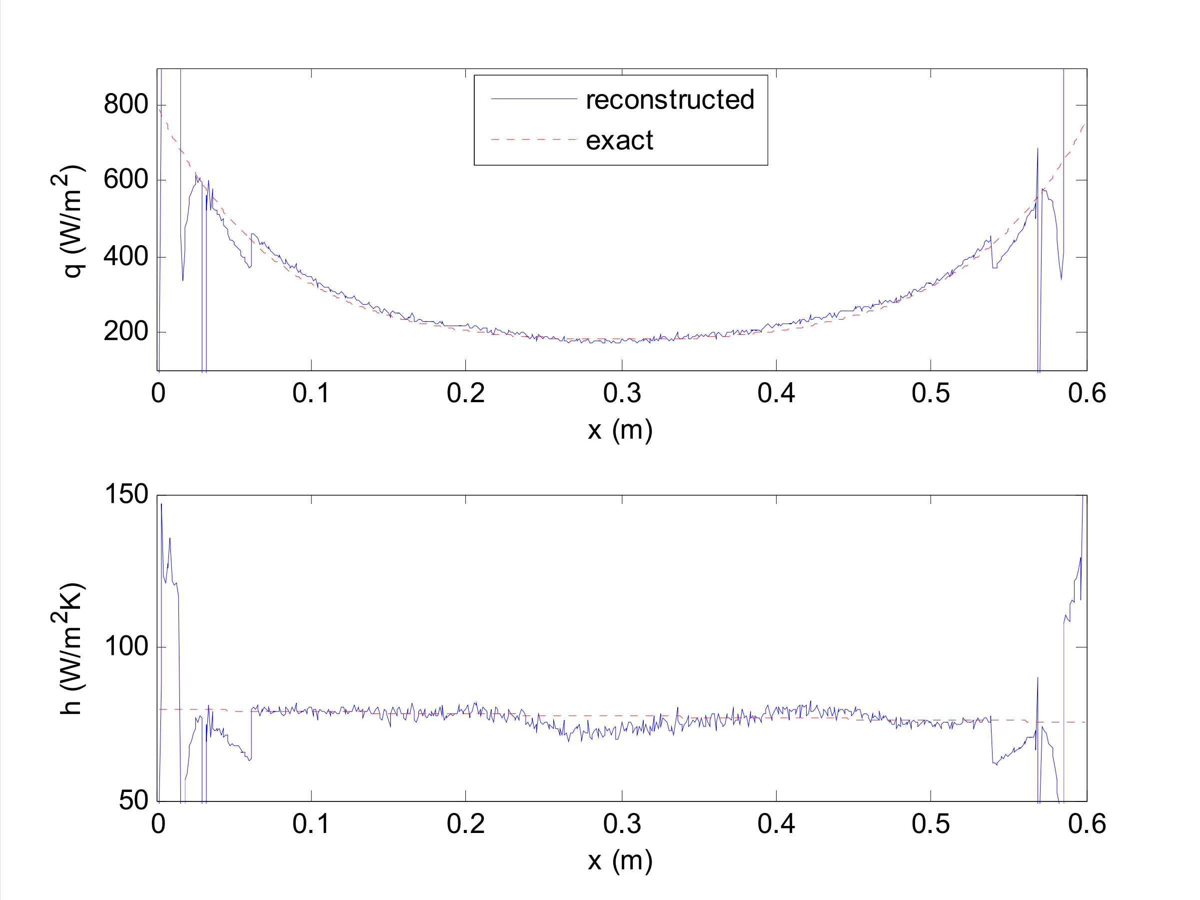
Results: Forced matching

good performance if a **spatial averaging filter** is applied to the results



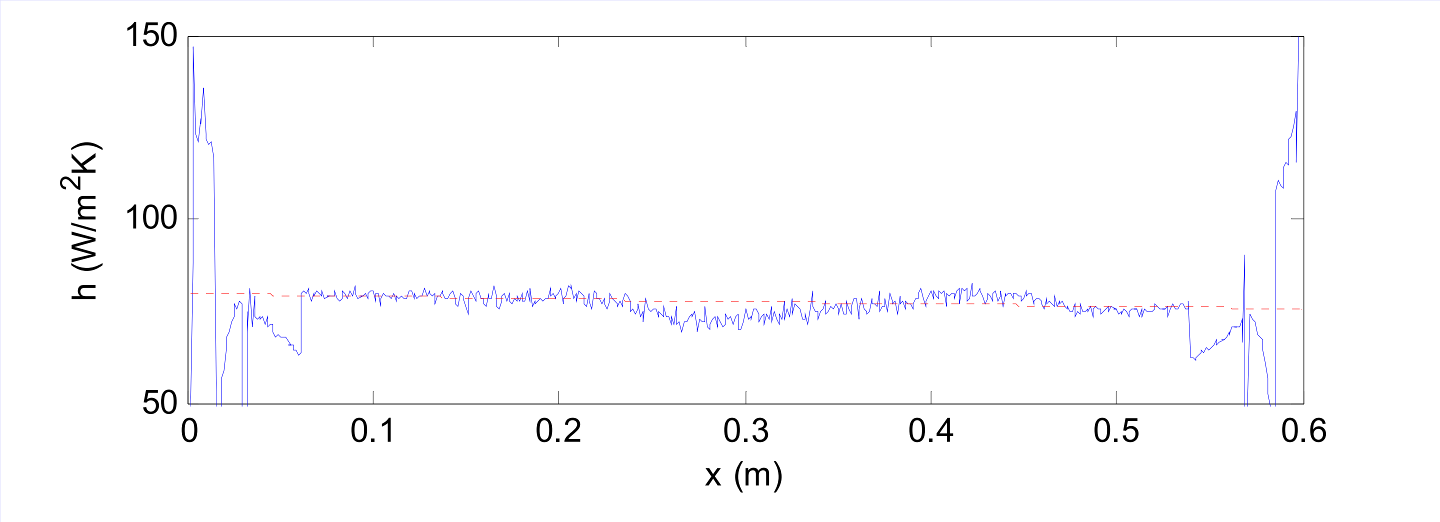
Surface heat flux and convective heat transfer coefficient distribution along the domain centerline

Results: Wiener filtering



Surface heat flux and convective heat transfer coefficient distribution along the domain centerline.

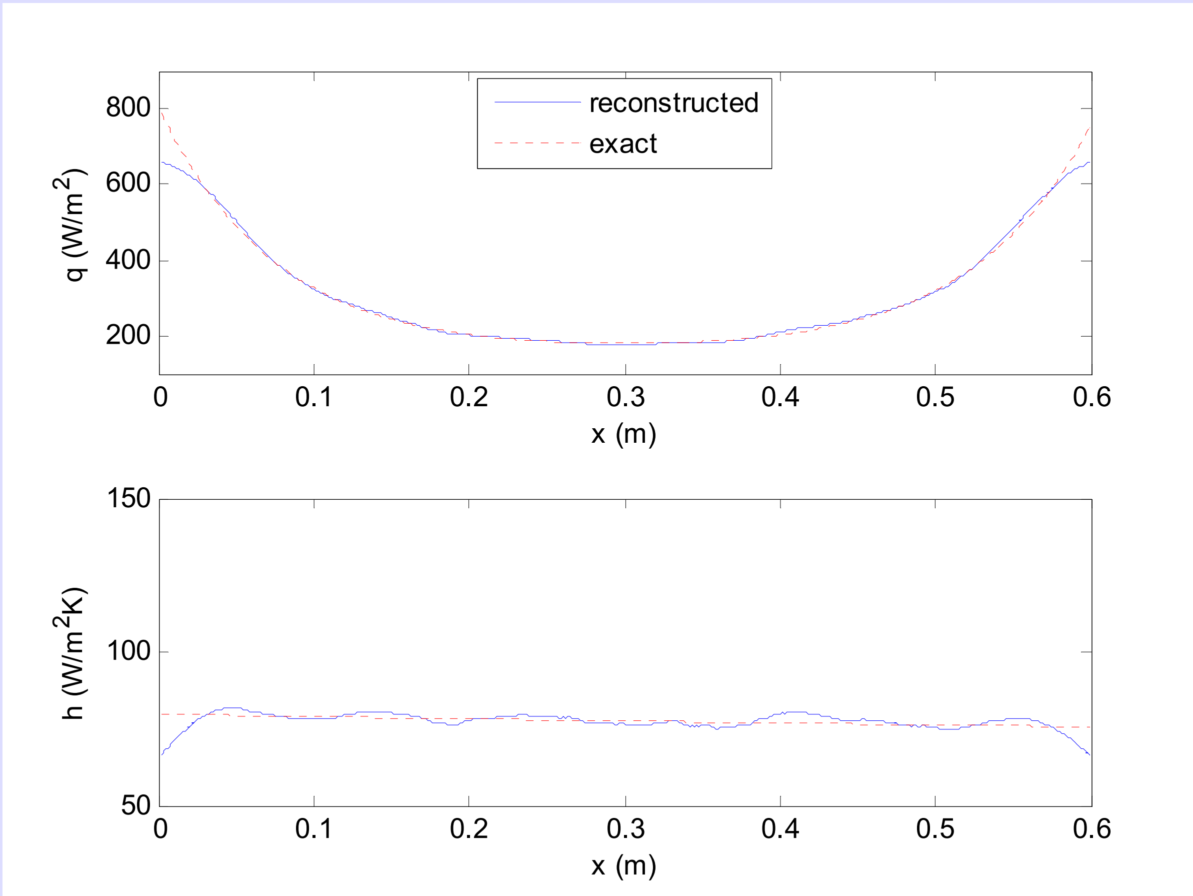
Results: Wiener filtering



- **PROS**: a good performance , limited computational resources required.

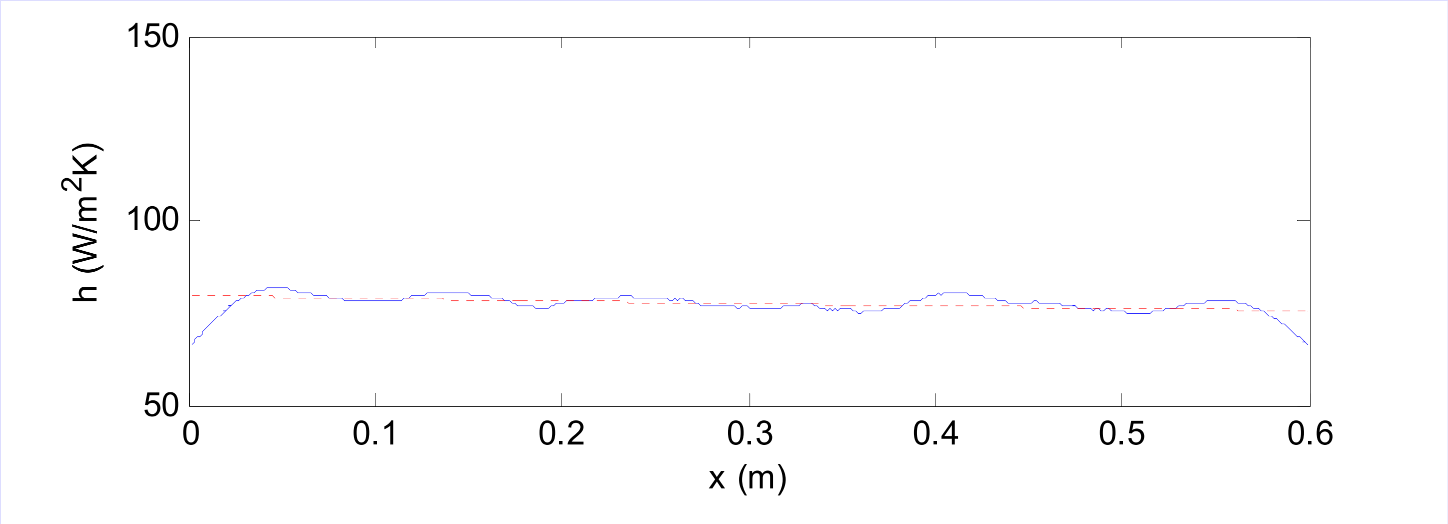
- **CONS**: confirms its well-known limit close to the border of the domain , there is not a clear criterion to chose window size of the filter.

Results: CGM



Surface heat flux and convective heat transfer coefficient distribution along the domain centerline.

Results: CGM



➢ **PROS**: the CGM, if compared to the other techniques, shows, with regards to this problem, a better performance.

➢ **CONS**: quite complex to implement, problems close to the domain's boundary.

CGM: domain's boundary

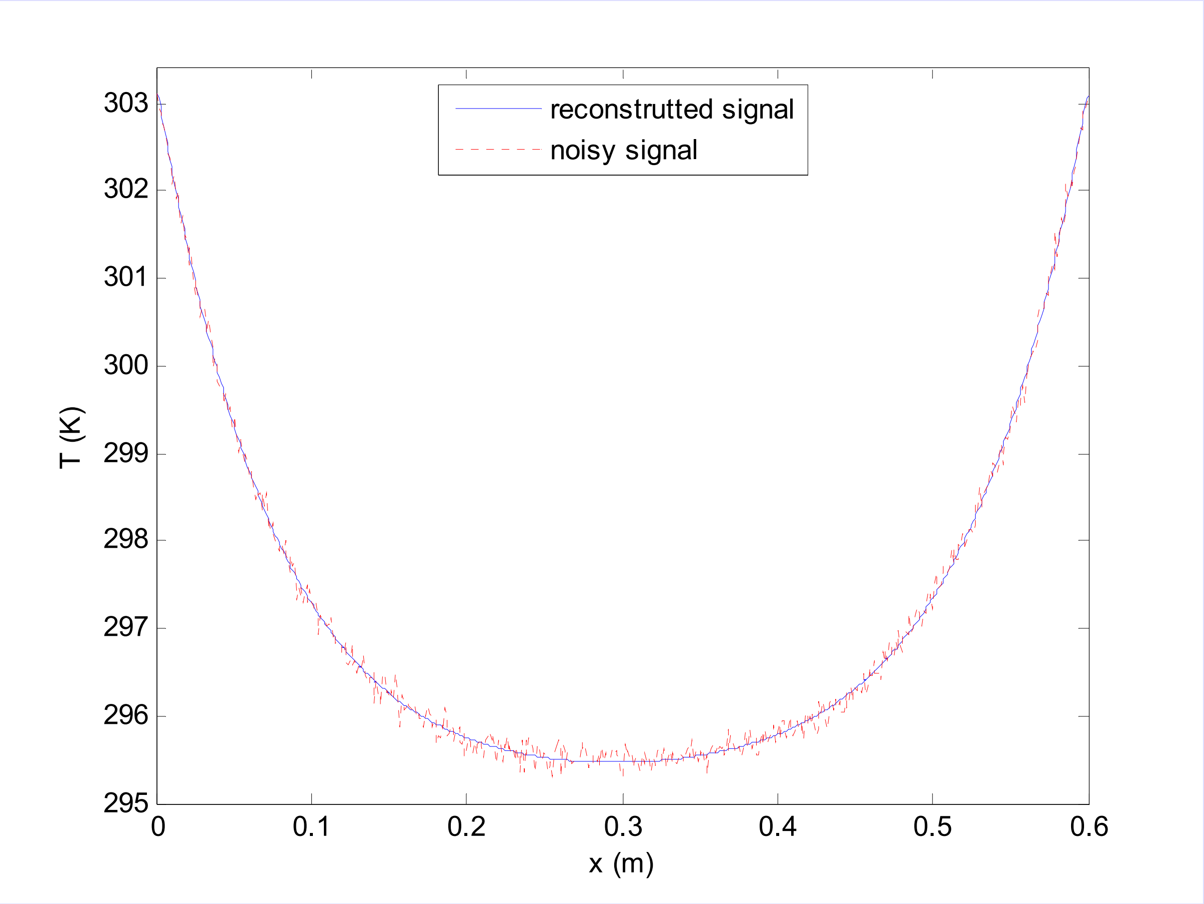
Although the **CGM**, here formulated with the adjoint problem, **converges rapidly** to the exact solution it has the major defect of being incapable of recovering the value of the unknown function close to the domain's boundary.

In fact the **vanishing of the gradient** of the conjugate gradient direction at the boundary forces the unknown function to be never updated along the iterative process.

Possible solutions:
- direct differentiation method (impracticable);
- supposing a certain distribution of the unknown function.

Luckily the error damps out by moving towards the central region

Results: CGM



Noisy and restore temperature distributions on the centreline of the plate.

Conclusions

The accuracy of the three solution methods has been quantified by means of an estimation error:

$$E = \frac{\left\| h_{estimated}(x_i, y_j) - h_{exact}(x_i, y_j) \right\|_{L_2}^2}{\left\| h_{exact}(x_i, y_j) \right\|_{L_2}^2} \cdot 100$$

Forced matching

E ≈ 14%

Forced matching (avarage)

E ≈ 0.5%

Wiener filtering

E ≈ 0.1%

CGM

E ≈ 0.02%

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